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K. KOMURASAKI
Nagoya University
Nagoya, Japan
and
G.V. CANDLER
University of Minnesota
Minneapolis, MN

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LAMINAR-TO-TURBULENT TRANSITION PREDICTION AROUND THE MUSES-C REENTRY CAPSULE

Kimiya KOMURASAKI
Department of Aerospace Engineering, Nagoya University
Chikusa, Nagoya 464-01, Japan
and
Graham V. CANDLER
Department of Aerospace Engineering and Mechanics, University of Minnesota
110 Union St. SE, Minneapolis, MN 55455

Abstract
Laminar-to-turbulent transition over an ablator heatshield of the MUSES-C reentry capsule has been analytically predicted. A two-equation turbulence model (k-ε model) was coupled with Reynolds averaged Navier-Stokes equations to reproduce the boundary flow transition. The low-Reynolds-number effect on the solid wall boundary was taken into account by modifying the Chien’s correction. The results show that the turbulence induced by turbulent-gas-injection from the ablator surface was multiplied in a boundary layer, and the flow transited to turbulence. The predicted transition point Reynolds number was $3 \times 10^4$ at the surface mass injection rate of 100 g/sm$^2$.

Nomenclature

$\varepsilon$ : specific heat at constant pressure
$d$ : Park’s mixing length on the turbulent wall
$E$ : total energy per unit mass
$F, G$ : flux vectors in $x$ and $r$ directions, respectively
$k$ : turbulence kinetic energy per unit mass
$K$ : Von Karman constant, 0.41
$p$ : static pressure
$Pr$ : Prandtl number
$q$ : heat flux
$R$ : nose radius, 0.2 m
$Re$ : Reynolds number, $\rho_\infty V_\infty R / \mu_\infty$
$Re_u$ : transition point Re, $\rho_\infty V_\infty s_u / \mu_\infty$
$s$ : distance from stagnation point along the body contour
$t$ : time
$T$ : temperature
$T_i$ : turbulent intensity, $T_i = \sqrt{\frac{2k}{3} \nu^2}$
$u, v$ : mean velocity components
$u', v'$ : fluctuating velocity components
$u_t$ : friction velocity, $u_t = \sqrt{\frac{\tau}{\rho}}$

$U$ : vector of conservative variables
$W$ : source vector
$x, r, \theta$ : cylindrical polar coordinates
$y$ : wall normal distance
$y^*$ : wall variable, $y^* = \frac{y u}{\nu}$
$\varepsilon$ : turbulence dissipation rate
$\gamma$ : specific heat ratio
$\kappa$ : thermal conductivity
$\mu_M$ : molecular viscosity
$\mu_t$ : eddy viscosity
$\nu$ : kinematic molecular viscosity
$\rho$ : density
$\tau$ : shear stress

Subscript
$I$ : inviscid
$SP$ : stagnation point
$T$ : turbulent
$tr$ : transition
$V$ : viscous
$W$ : wall condition
$\infty$ : free-stream condition

Introduction
The Institute of Space and Astronautical Sciences of Japan has proposed a space program MUSES-C, in which a spacecraft will rendezvous with an asteroid and bring samples of the asteroid material back to Earth by a reentry capsule. The spacecraft is scheduled to be launched in 2002 to rendezvous with an asteroid Nereus, and return in 2006. The atmospheric reentry speed will exceed 12 km/s, corresponding reentry Mach number is 42.

The laminar shock layer analyses conducted on this mission indicate that the maximum convective heat transfer rate on the capsule surface would be approximately 10 MW/m$^2$ and the radiative rate about 2 MW/m$^2$. In order to survive such an extremely high heating rate, the reentry capsule is shielded with a carbon-phenolic ablator. From a material test using an arc-heater, the recession rate of the ablator has been
estimated as 100 g/sm² at the heating rate of 10 MW/m². This surface-mass-flux corresponds to several % of free-stream mass-flux. Such a large amount of surface-mass-injection might induce turbulence even near the stagnation point of the capsule, resulting in a higher heating rate than predicted by laminar flow analyses.

Several experimental studies of laminar-to-turbulent transitions over a body with surface-mass-injection have been done in the United States so far, and those results have shown the transition.

Generally speaking, boundary flow transitions are hard to occur at the Re as low as the order of 10⁶, at which heating rate would be maximum during the MUSES-C reentry. However, Kattari's experiment predicted the transition Re = 3 x 10⁶. It implies that transition might occur in the MUSES-C mission. Such a low Re transition or "early" transition is thought to be caused by inhomogeneous and unsteady gas injection on the ablator surface.

The objective of this study is to analytically predict the transition Reynolds number and the maximum heating rate over the ablator of MUSES-C reentry capsule.

**Numerical model**

**Reynolds averaged Navier-Stokes equation**

Reynolds averaged Navier-Stokes equations coupled with the standard k-ε turbulent model over an axisymmetric blunt body are expressed as

\[
\frac{\partial U}{\partial t} + \frac{\partial (F_i - F_V)}{\partial x} + \frac{1}{r} \frac{\partial (r(G_i - G_V))}{\partial r} = W
\]

(1)

where

\[
F_i = \begin{bmatrix}
\rho
\rho u
\rho v
E
pk
pe
\end{bmatrix},
F_V = \begin{bmatrix}
0
\tau_{xx} + \tau_{xx}^T - 2/3 pk
\tau_{xy} + \tau_{xy}^T
(\tau_{xx} + \tau_{xx}^T/2 pk)u + (\tau_{xy} + \tau_{xy}^T)v + \kappa \partial_x T
(\mu_M + \mu_T) \partial_x k
(\mu_M + \mu_T/1.3) \partial_x \epsilon
\end{bmatrix},
\]

(2)

**Transport coefficients**

In the first-order turbulent closure model, the Reynolds shear stress \( \tau_R \) is expressed through the eddy viscosity, following Boussinesq's assumption:

\[
\tau_{ij}^R = \tau_{ij}^t - (2/3) \rho k \delta_{ij}
\]

(3)

\[
\tau_{ij}^t = \mu_t (\partial_i u_j + \partial_j u_i - (2/3) \nabla \cdot u \delta_{ij})
\]

(4)

In the standard k-ε model, the eddy viscosity is expressed as:

\[
\mu_t = 0.09 \rho k^2 / \epsilon
\]

(5)

Thermal conductivity coefficient can be defined through the Prandtl numbers. Assuming the laminar and turbulent Prandtl numbers for air flow to be constant as 0.72 and 0.9, respectively, total thermal conductivity is expressed as:

\[
\kappa = C_p (\mu_M/0.72 + \mu_T/0.9)
\]

(6)

In this study, neither chemical reactions nor real gas effects are taken into account, because we focus on transition mechanism through mechanical properties of the flow. Because of this, calculated post-shock temperature goes up to 80,000 K, which is several times higher than actual post-shock temperature. The molecular viscosity coefficient is obtained using an approximation for non-reacting air with a chemical composition frozen at standard conditions.

\[
\mu_M = 1.462 \times 10^{-6} T^{1/2} (1+112/T)
\]

(7)

**Low-Reynolds-number effect**

In order to accurately predict the boundary flow transition, limiting behavior of the fluctuating velocities approaching a solid boundary has to be considered. This effect has been expressed in following corrections as: 1) "non-isotropic dissipation" of turbulent energy, and 2) a "damping" effect on eddy viscosity, by Jones and Launder, Launder and Sharma, Chien, etc.

If the body surface is smooth and non-ablative, the fluctuating velocity satisfies the no-slip boundary condition and also satisfies conservation of mass. Therefore, from expanding the fluctuating velocity in Taylor series near a solid boundary, the tangential and
normal components \( u' \) and \( v' \) must behave as
\[
  u' = A(s,t)y + O(y^2), \quad v' = B(s,t)y^2 + O(y^3)
\]
respectively, as \( y \to 0 \).

The behavior of \( k \) and \( \varepsilon \) is deduced from these asymptotic variations of fluctuation velocities as:
\[
  k = u'^2 + v'^2 = A^2 y^2 + O(y^3),
\]
\[
  \varepsilon = \sqrt{u'^2 + v'^2} = A^2 + O(y)
\]
k decreases rapidly as \( y \to 0 \), while \( \varepsilon \) remains finite at \( y = 0 \). This is the "non-isotropic dissipation" of turbulent energy. To achieve this asymptotic consistency, the "wall" dissipation, \( \varepsilon_0 \) is added to the turbulent energy equation in most of the low-Reynolds-number corrections. In Chien's correction, \( \varepsilon_0 \) is expressed as:
\[
  \varepsilon_0 = 2\nu k/y^2
\]

To include the "damping" effect on eddy viscosity, Eq. (5) is often modified using a damping function, \( f_\mu \) as:
\[
  \nu_f = 0.09f_\mu\nu^2/\varepsilon
\]

In Chien's correction, \( f_\mu \) depends on a wall variable, \( y^* \):
\[
  f_\mu = 1 - \exp(-0.0115y^*)
\]

However, in the case of a charring ablator such as a carbon-phenolic one, there can exist finite velocity fluctuations even at \( y = 0 \) as schematically shown in Fig. 1, because the ablation gas is ejecting at subsonic speed through porous char remnant and allows disturbances to propagate from the boundary flow into the ablator. Assuming the limiting case when the surface fluctuation velocities perfectly couple with the ones of the boundary flow, the "non-isotropic dissipation", \( \varepsilon_0 \) is neglected in this study.

As for the viscosity "damping" effect, eddy viscosity should go to zero as \( y^* \to 0 \) as indicated in Eqs. (11) and (12). This effect preserves boundary flows from transition until \( Re = 10^5 \sim 10^6 \). Nevertheless, when the ablation gas is turbulent, eddy does not vanish on the surface. In order to take account of the eddy scale on the ablative surface, we define a new wall variable using a non-zero wall variable on the wall, \( y_w^* \) as:
\[
  y^* = y^*(u'/v') + y_w^*
\]

In this study, two cases are discussed; 1) the eddy scale is roughly larger than boundary layer thickness so that the damping effect is negligible:
\[
  y_w^* = \infty
\]

and 2) the eddy scale is given by the Park's wall mixing length model:
\[
  y_w^* = (d K(u'/v))
\]
\[
  d = \tau_e v_w
\]

Here, \( d \) is the Park's wall mixing length and \( v_w \) is the mean injection velocity. A time constant, \( \tau_e \) is an average time interval of fluctuation of injected flow. A reasonable value of the time constant can be taken to be \( \tau_e = 2 \times 10^{-3} \) s for carbon-carbon composite.

Although two more damping functions are usually added to the dissipation equation in the low-Reynolds-number corrections, they are neglected here because these effects are quite small for the turbulent-surface condition.

### Numerical scheme

FVS (Flux Vector Splitting) method is often used for calculating hypersonic blunt-body flow-fields, because of its robustness and suitability for use in implicit schemes, while it comes at a price of reduced accuracy due to numerical dissipation. The numerical dissipation is especially unfavorable to the turbulence calculation because it is essentially the calculation of turbulence-dissipation processes. On the other hand, FDS (Flux Difference Splitting) method is very accurate, though operation count and complexity are increased for complete linearization of flux formulas for implicit schemes.

Thereby, the AUSM (Advection Upstream Splitting Method) is employed in this study. This scheme combines the efficiency of FVS and the accuracy of FDS, and enables us to efficiently capture the hypersonic shock and to accurately evaluate boundary layer flows. The Gauss-Seidel line relaxation method is adopted to implicitly solve the nonlinear equations.

Figure 2 is a plot of a 120 × 30 grid used in this study. The master equations were discretized with a third-order upwind scheme using the MUSCL type interpolation. A grid convergence was obtained for heat flux distribution and for transition position.

### Boundary conditions

#### Free-stream

The free-stream Reynolds number is a key parameter for the amplification or suppression of eddy viscosity in the boundary layer. Rentry conditions at the flight altitude of 60 km and 76 km were tested corresponding to free-stream \( Re \) of 45000 and 6500, respectively. The free-stream conditions are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Free-stream conditions.</th>
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<tbody>
<tr>
<td>76 km altitude</td>
</tr>
<tr>
<td>density</td>
</tr>
<tr>
<td>temperature</td>
</tr>
<tr>
<td>Mach number</td>
</tr>
<tr>
<td>Reynolds number</td>
</tr>
<tr>
<td>60 km altitude</td>
</tr>
<tr>
<td>density</td>
</tr>
<tr>
<td>temperature</td>
</tr>
<tr>
<td>Mach number</td>
</tr>
<tr>
<td>Reynolds number</td>
</tr>
</tbody>
</table>
(In the MUSES-C mission, the heating rate is predicted to have a peak at approximately 60 km of altitude.) The free-stream is assumed turbulence free.

**Ablator surface**

The surface temperature is assumed identical to 4000 K and the local injection rates are assumed in proportion to the local heating rates as:

$$\rho w_0 = A q$$  \hspace{1cm} (17)

The injection-rate can be tuned by changing the coefficient $A$.

The ablation gas is injected perpendicular to the surface with a given turbulent kinetic energy. The turbulent intensity on the surface is specified to $0.03 \sim 0.3$.

Since the dissipation boundary condition, $\varepsilon_w$, doesn't give any distinguishable effect on the results, it was given so that $(0.09 pk / \varepsilon^2)_w = (\mu_m)_w$.

**Results**

**Case with $y_w^* = \infty$**

The calculated heating rate distributions over the capsule surface are shown in Fig. 3. In the case $Re = 45000$, the stagnation-point heating is decreased with an increase in surface injection rate due to the heat-blockage effect. Every plot shows an increase in heating rate in the downstream region due to the laminar-to-turbulent transition, except for the no-injection case. At the injection rate of 80 g/sm$^2$, heating rate has a peak at $s = 12$ cm, and the maximum rate was approximately 25% higher than the stagnation-point one.

In the case $Re = 6500$, the flow stayed laminar for any injection rate.

Figure 4 shows the profiles of mean velocity and turbulence kinetic energy in the case $Re = 45000$. The plots are enlarged in the surface normal direction by a factor of five. Turbulence kinetic energy is amplified in the stream-wise direction in the middle of boundary layer.

Figure 5 shows the distributions of the eddy viscosity and molecular viscosity. These plots are also enlarged in the surface normal direction. The eddy viscosity surpassed the molecular viscosity in the downstream region.

The peak eddy viscosity along the wall normal lines is plotted in the stream-wise direction in Fig. 6. It was normalized by the local molecular viscosity. The nonlinear amplification of the eddy viscosity indicates the transition. The transition point shifts upstream with an increase in the surface injection rate.

**Case with $y_w^* = (d/K)(u_*/\nu)$**

Figure 7 shows the distributions of the heating rate and the peak eddy viscosity along the surface at the injection rate of 80 g/sm$^2$. Comparing with Figs. 3 and 6, transition delayed due to the viscosity damping effect. The flow stayed laminar at the injection rate less than 80 g/sm$^2$.

Figure 8 shows the heating rate distributions for various injection rates. Transition occurs earlier with larger injection rates.

The distance from the stagnation point to the transition point is plotted for various surface turbulent intensity in Fig. 9. It shows a weak dependence on the surface turbulent intensity condition. This is due to the fact that the kinetic energy of the injection gas is much smaller than that of main flows, and the turbulent energy in the vicinity of surface is determined through the diffusion, $(\mu_\lambda + \mu_m) \nabla k$ from the region where the flow is turbulent.

Owing to the same reason, the calculation is insensitive to the dissipation condition on the surface, $\varepsilon_w$ as mentioned above.

If $T_i = 0$ on the surface, the flow stays laminar at any injection rate because there is no seed of turbulence.

Figure 10 shows a streamline of the gas injected near the stagnation point. The seed of turbulence given near the stagnation point is carried along the streamline and is gradually amplified by the work of the main flow against the Reynolds stress. The streamline is separated from the surface and swelled into the middle of boundary layer by successive gas injection in the downstream region of the surface. With the small injection rate, the streamline stays in the region where the viscosity damping is strong, while with the large injection rate, the streamline is pushed into the region where $Re$ is relatively high and damping effect is weak, resulting in the transition.

**Injection rate and transition point $Re$**

Figure 11 shows a comparison between the Kaattari's transition experiment and the present prediction. The injection rate normalized by free-stream mass-flux is on the abscissa and the transition point $Re$ is on the ordinate.

In the case with $y_w^* = \infty$, calculated results show earlier transition than the experimental prediction. Transition occurs even at the quite small injection rate. On the other hand, in the case with $y_w^* = (d/K)(u_*/\nu)$, transition doesn't occur at the injection rate less than 80 g/sm$^2$. The calculated transition envelope is well agree with the experimental result.
Discussions

The mechanism for the early transition induced by surface mass injection would consist in following two points.

1) The reduction of viscosity damping due to the surface turbulent condition.

2) The swelling of stream-lines of the ablation gas by successive gas injection in the downstream region of the surface.

Both of them assist the amplification of eddy viscosity in boundary flows. These effects strongly depend on the ablation rate on the surface. That is consistent with Kaatari’s experiment.

Since the predicted transition point Re shows good agreement with the experimental results, it can be concluded that this numerical model well represents the early transition mechanism due to surface mass injection.

References


Fig. 1 Schematic of velocity fluctuations on a charring ablator.
Fig. 2 Plot of a $120 \times 30$ grid.

Fig. 3 Heating rate distributions on an ablative surface. $y_w^* = \infty$, $T_i = 0.1$.

(a) $Re = 45000$

(b) $Re = 6500$

Fig. 4 Profiles of mean velocity and turbulent kinetic energy. $Re = 45000$, $y_w^* = \infty$, $T_i = 0.1$, $(\rho_w \nu_w)_{sp} = 80$ g/sm$^2$. Enlarged in the wall normal direction by a factor of five.

Fig. 5 Profiles of eddy viscosity and molecular viscosity. $Re = 45000$, $y_w^* = \infty$, $T_i = 0.1$, $(\rho_w \nu_w)_{sp} = 80$ g/sm$^2$. Enlarged in the wall normal direction by a factor of five.
Fig. 6 Eddy viscosity distribution. $Re = 45000$, $y_w^* = \infty$, $T_i = 0.1$.

Fig. 7 Distributions of heating rate and eddy viscosity. $Re = 45000$, $y_w^* = (d/K)(u_t/v)$, $T_i = 0.1$.

Fig. 8 Heating rate distributions on an ablator surface. $Re = 45000$, $y_w^* = (d/K)(u_t/v)$, $T_i = 0.1$.

Fig. 9 Surface turbulent intensity and transition distance from stagnation point. $Re = 45000$, $y_w^* = (d/K)(u_t/v)$.

Fig. 10 A streamline of the gas injected near stagnation point and contours of $(\mu_t/\mu_m)$. $Re = 45000$, $y_w^* = (d/K)(u_t/v)$, $T_i = 0.1$, $(\rho w v^w)_{\exp} = 80 \text{ g/sm}^2$. Enlarged in the wall normal direction by a factor of 20.

Fig. 11 Transition point $Re$ and injection rate ratio.