LSD Termination Conditions and Post-Shock Electron Density

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Electron number density behind a laser supported detonation (LSD) wave was measured by the two-wavelength Mach Zehnder interferometry. As a result, the density was maintained at about 2×10^24/m^3 during the LSD regime in 2D and quasi-1D blast wave expansion conditions. It suggests that this is a critical density necessary to sustain LSD in atmospheric air by CO2 lasers. Temperature was estimated at 12000~15000 K assuming local thermal equilibrium. Deduced absorption length was at 0.10~0.13 mm. It suggests that the laser was absorbed in a very thin region of the plasma layer.

Nomenclature

\begin{align*}
c & = \text{velocity of light} \\
d & = \text{thickness of phenomena} \\
e & = \text{electron charge} \\
h & = \text{number of fringe shift} \\
k_B & = \text{Boltzmann constant} \\
K & = \text{relative refractive index} \\
l & = \text{thickness of a plasma layer behind a LSD wave} \\
m_e & = \text{electron mass} \\
n & = \text{number density} \\
N & = \text{refractive index} \\
r & = \text{laser beam spot size} \\
U_i & = \text{ionization energy} \\
Z & = \text{partition function} \\
\varepsilon_0 & = \text{electrical permittivity of vacuum} \\
\lambda & = \text{wavelength of the light source}
\end{align*}

Subscripts

\begin{align*}
n, i, e & = \text{neutral particle, ion and electron}
\end{align*}

I. Introduction

When a high power laser beam is focused in air, breakdown occurs and a blast wave is generated. Electrons, emanated by the breakdown, absorb the laser energy and are accelerated. Then surrounding air is heated and the pressure is increased by the collisions between electrons and heavy particles. In such processes, laser energy is converted into a shock wave. The Repetitively Pulsed (RP) laser propulsion,[1-3] proposed as a low-cost launch system, is one of the devices in which a shock wave induced by a laser beam is playing an important role: A high power laser pulse is beamed toward a vehicle and thrust is produced by a reflection of the induced shock wave.
Laser energy is absorbed by the plasma in a laser supported detonation (LSD) regime at the laser intensity higher than 1-10 MW/cm². In this regime, a plasma front adjoins a shock wave. Because plasma is isometrically heated by a laser beam in a layer close behind the shock wave, the shock wave is driven further. After the plasma region has been separated from a shock wave, plasma is heated isobarically and the shock wave expands almost adiabatically. Therefore, no contribution is expected to the thrust work. Because the LSD regime inevitably terminates with the decay of the laser intensity on the LSD wave, the LSD termination condition is very important for the optimal design of the devices such as the PR laser propulsion.

Figures 1 and 2 respectively show the evolution of a shock wave and plasma induced by a line-focused laser beam (the 2D LSD condition,) and that with a confinement nozzle (the quasi-1D LSD condition) generated by a TEA CO₂ laser. As indicated in Fig. 3, 90% of the energy has been irradiated in 3 μsec.

Figure 4 shows the temporal variation of the shock wave and the plasma front displacements on the laser axis. As seen in the figure, the shock wave and the plasma front began to separate, namely the LSD terminated at 1.2 μsec in the 2D condition and at 1.8 μsec in the quasi-1D condition after the breakdown. [5]

Raizer derived a LSD theory: the LSD terminates at a certain plasma layer thickness l which expands with time due to the lateral enthalpy loss from the LSD region as schematically shown in Fig.5. The balance of lateral energy loss and energy input on the LSD wave had been characterized using a ratio of the lateral surface of the cylindrical plasma layer, \( S_{\text{side}} = 2\pi rl \) to the area of the incident plane of the laser, \( S_{\text{fron}} = \pi r^2 \). Raizer's termination criterion was \( S_{\text{side}} / S_{\text{fron}} \approx 8 \). [6]

However, the LSD termination condition should be characterized using plasma parameters behind the LSD wave, because that would be very useful for optimization of device design in various scales, atmospheric pressures and laser wavelengths. In this study, electron number density behind the LSD wave was measured by the two-wavelength Mach Zehnder interferometry to find out a direct relationship between plasma parameters and the LSD termination. In addition, the LSD confinement effect on plasma parameters was examined comparing the quasi-1D and 2D cases.
II. Laser Focusing Apparatus

The plasma was produced by a TEA CO$_2$ pulse laser whose nominal energy is 10 J. Its power history has been already shown in Fig.3. The beam profile is the Gaussian distribution on the horizontal axis and trapezoidal distribution on the vertical axis. The laser beam cross-section is a $30 \times 30$ mm square.

The laser beam was focused in the horizontal direction using an off-axis line-focusing parabolic mirror. Its focal length is 48 mm along the optical axis. It reflects an incident laser beam by 90 degrees along the optical axis into the focus. The line width on the focal line is estimated at 0.25 mm.

Because radiation from the plasma seriously disturbed the fringes when the plasma thickness was 30 mm, the laser beam was partially intercepted by a slit to slice the LSD by 2 mm as shown in Fig. 6. The top and bottom of the test section were closed with two glass plates to avoid the edge-effect. The shock propagation speed and the LSD termination timing were identical in the 30 mm thick and 2 mm thick LSD waves.

Figure 5: The conceptual rendering of Raizer’s LSD termination theory.

Figure 6: Laser focusing apparatus for LSD wave visualization.
III. Two wavelength Mach-Zehnder interferometry

The Mach-Zehnder interferometry is a non-intrusive and quantitative visualization method, which responds fast enough to acquire non-steady phenomena like a shock wave. The particle number density is measured by counting the shift of fringes. However, the electron density can’t be measured by conventional Mach Zehnder interferometry because the shift is a superposition of electron and heavy particle contributions.

Using two probe beams, having different wavelength, the electron number density can be estimated because electrons have a wavelength dependency. The measurement system is shown in Fig.7. “A” in the Fig.7 is different from the conventional Mach Zehnder interferometry. The laser beam was divided into two beams by Beam splitter #4, and projected on an ICCD camera through band-pass filters (633±1 nm or 532±1 nm).

![Figure 7: The two-wavelength Mach-Zehnder interferometry system.](image)

Figure 8 shows the Mach Zehnder interferometry images of the LSD. The laser started to irradiate at $t = 0 \mu$sec. The displacement of the fringes on the laser axis from the baseline is measured. The baseline fringes are interpolated from the side regions not disturbed by the blast wave.

![Figure 8: Two-wavelengths Mach Zehnder interferometry images, left: 532nm, right: 633nm.](image)

The difference in the refractive index between the reference and the object $\Delta N$ is expressed as

$$\Delta N = \frac{\hbar \lambda}{d}. \quad (1)$$

The relation between $\Delta N$ and number densities in the object is expressed as

$$\Delta N = K_e n_e - (K_n n_n + K_p n_p + K_i n_i). \quad (2)$$

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Here, \( n_0 \) is the reference neutral particle number density. The relative refractive indices of neutral particle and ion is approximately constant; 
\[
K_n = 1.1 \times 10^{-20} \text{ [m]}^3, \quad K_i = 7.4 \times 10^{-30} \text{ [m]}^3.
\]
On the other hand, \( K_e \) is dependent on wavelength of the probe laser\(^\text{[7]}\), which is expressed as
\[
K_e = -\frac{e^2}{8\pi \epsilon_0 c m_e} \lambda^2
\]
Using two wavelengths denoted by 1 and 2, \( n_e \) can be deduced as
\[
n_e = \frac{\Delta N_1 - \Delta N_2}{K_e(\lambda_2) - K_e(\lambda_1)}
\]

**IV. Results and Discussion**

Figure 9 shows the electron number density distribution on the laser axis deduced from the fringe shift using Eqs. (1)-(4). In the LSD regime, a peak of the electron density was clearly recognizable. The peak electron density was about \( 2 \times 10^{24} \text{[m}^{-3}\text{]} \) in the LSD regime in both cases. Then, the peak density became smaller after the LSD was terminated.

Figure 9: Temporal variation of the electron number density distribution on the center axis.

(a) 2D

(b) quasi-1D

The thickness of a plasma layer \( l \) was defined as the distance from the shock wave to the location where the electron number density becomes \( 1/e \) of the peak density. Thus \( S_{\text{side}} / S_{\text{front}} \) was derived as Fig.10. The error bars show the deviation in multiple measurements. \( S_{\text{side}} / S_{\text{front}} \) was proportionally increased with time. As seen in the figure, \( S_{\text{side}} / S_{\text{front}} = 0.7 \pm 0.2 \) at the LSD termination (\( t = 1.2 \mu\text{s} \)) timing. This value is much smaller than the Raizer’s criterion (\( l/r = 4 \), i.e. \( S_{\text{side}} / S_{\text{front}} \approx 8 \)).
Figure 11 shows the relationship between the peak electron density and laser intensity on the LSD wave. The laser intensity decreases with time because laser power decays and $r$ on the LSD wave increases. In both cases the peak electron density remained at about $2 \times 10^{24}\text{m}^{-3}$, and it dropped rapidly after the LSD was terminated. It suggests that the electron density of about $2 \times 10^{24}\text{m}^{-3}$ is a critical density necessary to sustain LSD in atmospheric air by CO$_2$ lasers. The cut-off electron density of a CO$_2$ laser is $9.94 \times 10^{24}\text{m}^{-3}$, which is five times as large as this density.

Assuming local thermal equilibrium, electron temperature was calculated using the Saha equation.

$$\frac{n_e}{n_i} = \frac{Z^2}{Z^2} \left( \frac{2 \pi m k T}{\hbar^2} \right)^{3/2} e^{-\frac{E_i}{kT}} \approx 2.419 \times 10^{22} T^{3/2} e^{-\frac{15}{kT}}$$

The density of the heavy particles was also estimated from the fringe shift of the two-wavelength Mach-Zehnder interferometry. As a result, electron temperature was at about 12000~15000 K and the degree of ionization was at 1~3% at the electron density peak.

The absorption length was calculated at 0.10~0.13 mm from obtained electron density and temperature. Here, only inverse Bremsstrahlung radiation was considered. It suggests that the laser was absorbed in a very thin region of the plasma layer.

V. Conclusion

- Electron densities behind the 2D LSD and quasi-1D waves were found maintained at about $2 \times 10^{24}\text{m}^{-3}$ during the LSD regime. It suggests that this is a critical density necessary to sustain LSD in atmospheric air by CO$_2$ lasers.
- At the LSD termination, $S_{side}/S_{front}$ was $0.7 \pm 0.2$, which is much smaller than that of Raizer’s prediction.
- Temperature was estimated at 12000~15000 K and the absorption length was at 0.10~0.13 mm. It suggests that the laser was absorbed in a very thin region of the plasma layer.

References